

# Homework # 3 – chapter 2 Eugene Hecht

1. About the wave-particle duality of light, Hecht states that the essential feature of a **particle** is its location: “it exists in a **well defined small** region of space”; while the essential feature of a wave is its non-localization: “a self-sustaining **disturbance of a medium**, transporting **energy and momentum**”;
  - a. What is the problem with the first statement for the particle (Hint: what is the physical definition of space)? Is this consistent with relativity?
  - b. What medium is disturbed in the case of light? What does this view imply for the physical definition of space?
2. According to Quantum Electrodynamics (QED) a photon is a quantum of energy  $E = h\nu = pc$ , where  $\nu$  is a frequency,  $h$  is Planck constant,  $p$  is the linear momentum related with the photon and,  $c$  is the velocity of light;
  - a. Show that the unity of  $[h] = \text{J} \cdot \text{s} = [L]$ , where  $L$  is an angular momentum
3. According to the Broglie, a particle of matter with momentum  $p = mv$  can also be expressed as a wave;
  - a. Find the expression for the wavelength of this particle and determine its value for an electron moving around the nucleus (note that applying the model of Bohr, the velocity  $v = 2.19 \times 10^6 \frac{\text{m}}{\text{s}}$ , which is only 1% of the velocity of light).
  - b. What is the consequence of the Broglie view of particle for the physical definition of space (hint: consider the definition of Hecht for a wave)?
4. In an atom, the level of energy  $n$  of an electron in turn of the nucleus is quantized—which means that the energy  $E_n = -\frac{hcR}{n^2}$ , where  $R$  is an empirical constant;
  - a. Shows that this implies that the quantization of energy is in fact equivalent to the quantization of angular momentum of the particle  $\frac{L}{h} = \frac{n}{2p}$
  - b. What does this suggest for the nature of the photon (hint: can you separate the photon from matter)?
5. In QED, particles interact through fields: “the field is the particle and the particle is the field”; The above is also true in General Relativity—Gravity, the curvature of space-time, is the field that describes the interaction between mass;
  - a. What does this tell you about the physical nature of space?
  - b. Do you find any inconsistencies in the duality wave-particle of light (hint: the answer should be no, but the question is more if the demonstration is clear for you)?

6. What is the effect of using as a waveform the function  $\mathbf{y}(x, t) = f(x \pm vt)$ ?
7. Derive and explain the differential equation of wave as encountered by d'Alembert (be careful to include all details of demonstration);
  - a. Explain in physical terms what it means that this equation is partial, linear, homogeneous, 2<sup>nd</sup> order?
  - b. What is the consequence of this equation for the definition of a wave?
8. An harmonic wave as the form  $\mathbf{y}(x, t) = A \sin(kx - \omega t + \mathbf{e})$ 
  - a. What are  $k$ ,  $\omega$  and  $\mathbf{e}$ ?
  - b. Show that  $\left(\frac{\partial x}{\partial t}\right)_y = \pm \frac{\omega}{k} = \pm v$  and explain what it means
  - c. Why harmonic waves are so important (hint: think Fourier)?
9. What is the physical definition of a Wavefront?
10. What are the differences between an ideal wave and a real one?
11. What is the principle of superposition? What physical properties must be assumed for the medium where the deformation takes place? What is the importance of the phase in this process?
12. Show that the plane wave  $\mathbf{y}(\vec{r}) = A e^{i\vec{k} \cdot \vec{r}} = A e^{i\vec{k} \cdot \vec{r}} e^{i k x}$  and deduce the relation between  $k$  and  $\mathbf{l}$
13. Starting with the Cartesian form of the Laplacian show that the wave equation for a spherical wave is  $\frac{\partial^2}{\partial r^2}(r\mathbf{y}) = \frac{1}{v^2} \frac{\partial^2}{\partial t^2}(r\mathbf{y})$ ; What are the main characteristics related to the solution of this equation (hint: Amplitude, Wavefronts, and Wavefront far from source)?