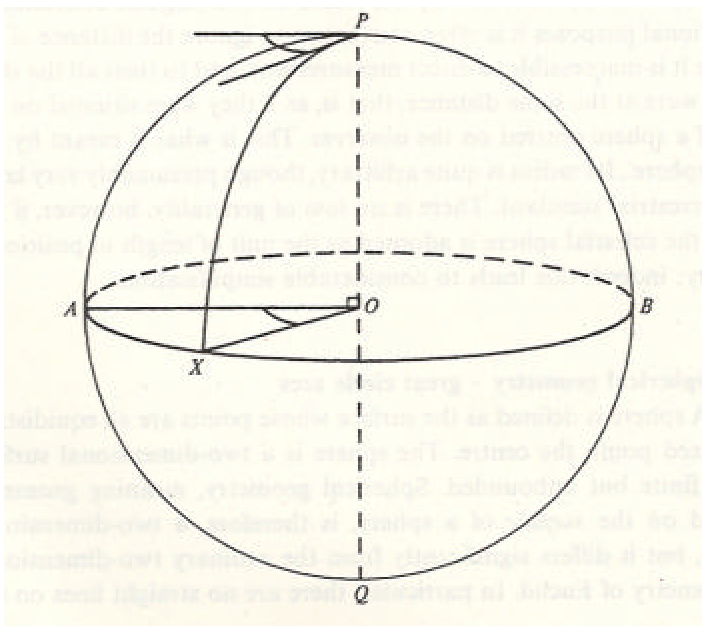
Basic notions of spherical Astronomy

Sphere = surface whose points are all equidistant from a fixed point = center

- 2D surface finite but not bounded
- Spherical geometry = 2D geometry
- No straight lines on surface of sphere – equivalent = arc of great circles

Great circle = any plane passing through center of sphere intersect the sphere in a great circle



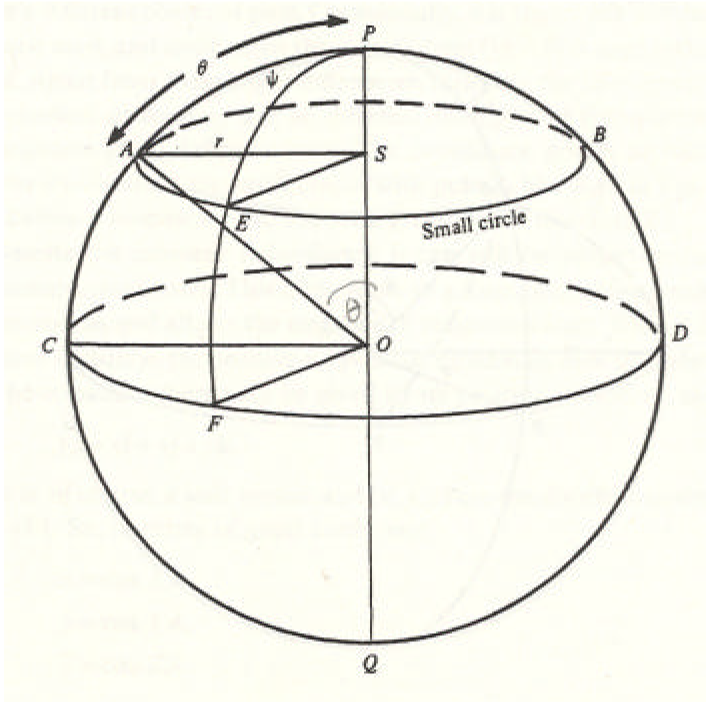
Poles = two extremities of diameter of sphere perpendicular to that of great circle

Consider 2 points on the sphere *A* and *X*

- Points *A*, *X* and center *O*, together define a unique plane that can be great circle
- If *R* is the radius of the sphere ($R = 1$) the **length of arc** = the angle in radians subtends at the center of the sphere
- Two arcs: AX and $ABX = 2p - AX$ (practical use smaller one)

Spherical angle between two intersecting great circle arcs = the angle between their planes: $APX = A\hat{O}X = AX$

Plane intersecting sphere not passing through center = **small circle**

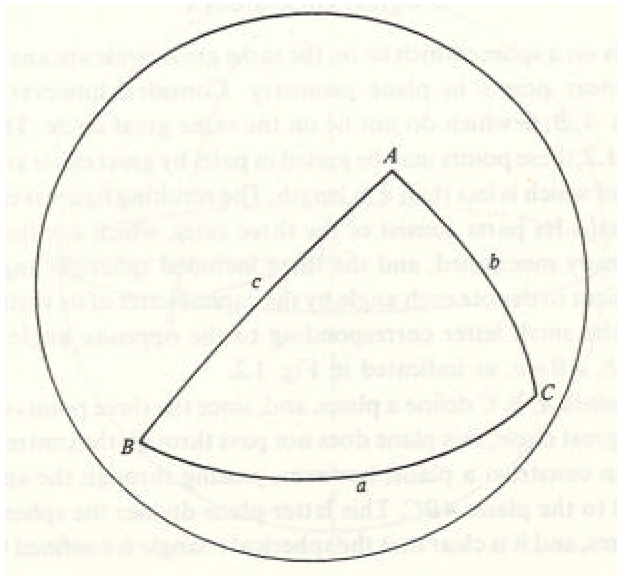


Radius of small circle: $r = \sin q$, where $r = AS$ and $q = A\hat{O}S$”

Small circle arc: $AE = rA\hat{S}E = y \ \sin q$, where $y = C\hat{O}F = APE$

NOTE: AE is not a geodesic $\Rightarrow$ not the shortest distance between two points

Spherical trigonometry



Consider three points *A*, *B* and *C* on surface of the sphere but not on same great circles

Spherical triangle: 3 sides = great circles AB, BC and AC including spherical angles a, b and c such that $AB = c$, $CA = b$ and $BC = a$

- Three points A, B and C define a plane
- Each angle $> 180^\circ$ - may have 1, 2, or 3 right angles $\Rightarrow$ sum of angles $> 180^\circ$
- All points of spherical triangle lie between 0 and 180° - confine to first quadrant $\Rightarrow$ sine terms is ambiguous but not cosine term

Basic formulae:

1. **Cosine formula**: $\cos a = \cos b \cos c + \sin b \sin c \cos A$
2. **Sine formula**: $\dfrac{\sin A}{\sin a} = \dfrac{\sin B}{\sin b} = \dfrac{\sin C}{\sin c}$
3. **Analogue formula**: $\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$
4. **Four parts formula**: $\cos a \cos C = \sin a \cot b - \sin C \cot B$

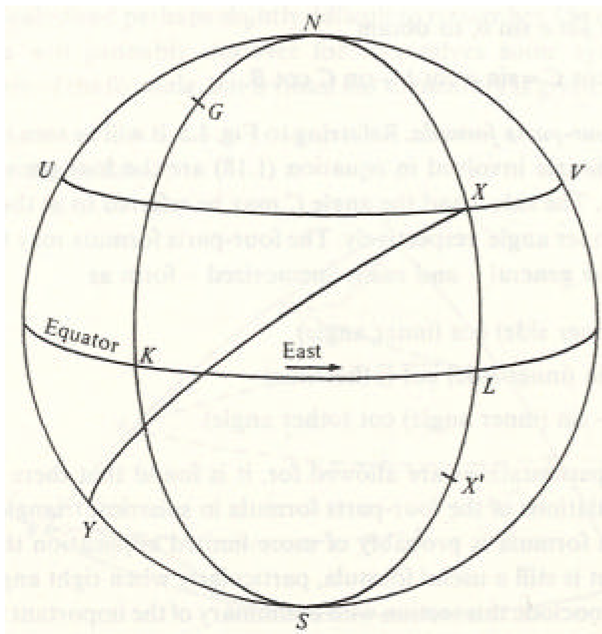
Terrestrial coordinates

Earth = rotating sphere

Equator = great circle with N and S has its poles

Meridian of longitude = any great circle perpendicular to equator

Prime Meridian = origin of system – $NGKS$ - international agreement 19th century – ***Royal Greenwich Observatory***



Position on Earth determined by great circle NX and spherical angle GNX

- **Latitude** $f = 90^\circ - NX$ where NX is the **co-latitude**
- **Longitude** $l = GNX$

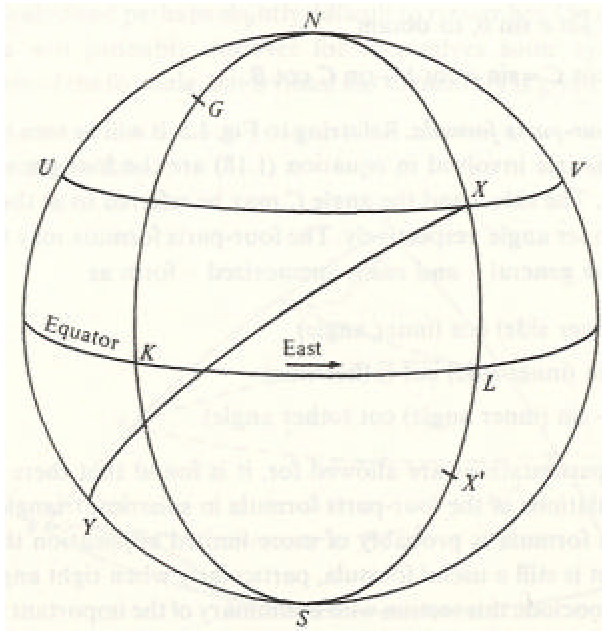
Extending NX to complete meridian $NXLS$ all points have same longitude **– parallel of longitude**

Construct small circle UVX through X with N and S as poles and all the points have same latitude $\Rightarrow$ **parallel of latitude**

Parallel of latitude + parallel of longitude = coordinate grid

- Latitude 0° to 90° North and South (negative to the south)
- Longitude 0° to 180° East to West – longitude positive in anticlockwise direction

Distance between two points



$X(f, l)$ and $Y(f', l')$ - shortest distance = geodesic – part of spherical triangle NXY

- $NX = 90^\circ - f$
- $NY = 90^\circ - f'$
- $GNX = l$ and $YNG = -l'$

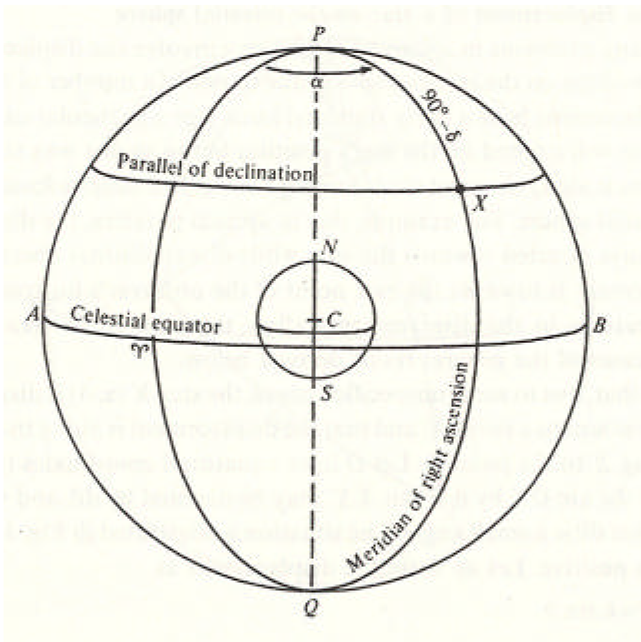
As an angle, the distance: $\cos XY = \sin f \sin f' + \cos f \cos f' \cos(l - l')$

To convert in km: transform in circular measure and multiply by radius

Nautical mile: length of great circle arc which subtends an angle of 1 arcminutes at Earth center

- Length XY = arc XY in arc minutes
- 1 nautical mile ≈ 1.855 km

Celestial sphere



Extension of Earth sphere at very large radius

Celestial equator = projection of equator on the sky

Coordinates: **right ascension** + **declination**

Parallel of declination = small circle parallel to celestial equator

Meridian of right ascension = semi great circle for point X

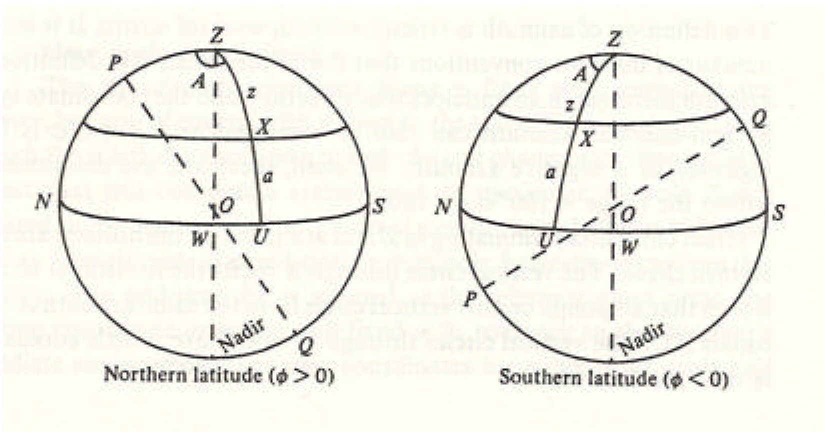
Declination: $d = 90^\circ - PX$
Right ascension: $a = \Upsilon PX$

- Where Υ is a fixed point on the celestial equator – marks Sun's position at **northern vernal equinox** (when Sun crosses celestial equator from South to North around 21 of march)
- a increases to the East
- Ascension given in hour angle: $24h = 360^\circ$
  - $1h = 15^\circ$, $1\min = 15'$ and $1s = 15''$
  - $1^\circ = 4\min$, $1' = 4s$ and $1'' = \frac{1}{15}s$

Conversion factor
1radian = $57^\circ 17' 45'' = 206265''$
For small angle $\sin q \approx q$, $\cos q \approx 0$ and $\tan q \approx q \Rightarrow \sin 1'' = \frac{1}{206265}$ radian

Alt-azimuth system

Celestial sphere centered on observer



Most natural reference direction = vertical (defined by Earth gravity)

Intersection upward with celestial sphere = **zenith** Z (opposite = **nadir**)

Celestial horizon = great circle which has Z and nadir as poles

- Separated celestial sphere in two hemispheres – upper visible and lower hidden from observer

Due to diurnal rotation of Earth in West-East direction (axis of rotation PQ) – celestial sphere apparent motion from East to West

Great circle ZP intersect horizon at North and South – great circle at right angle past by West and East: **cardinal points of horizon** $NESW$

Alt-azimuth system = local system – based on Z (reference ZN or ZP)

If X is position of object on celestial sphere:

- **Zenith distance** $z = ZX$
  - **Altitude** $a = 90^{\circ} - z$
- **Azimuth** $A = PZX$ - azimuth west - increases anticlockwise (right handed system)

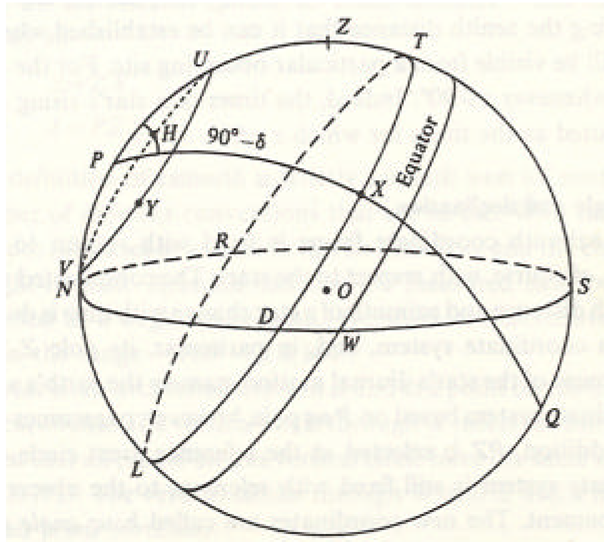
Vertical circle = Great circle arc terminating in Z at a point on horizon - ex. ZXU – all points have same azimuth NU

- **Prime vertical**: vertical circle through E and W

Small circle with Z as pole – all points have same altitude or zenith distance

- **Parallel of altitude**

If f is the geometric latitude of the site, then $PZ = 90^{\circ} - f$



Alt-azimuth system = local $\Rightarrow$ determine if and when an object on the celestial sphere is visible

Coordinates:

- **Declination** $d = 90^{\circ} - PX$
- **Hour angle** $H = ZPX$ (or HA)

The arc PX is the **north polar distance** (NPD)

- Semi great circle terminating at pole PXQ is **meridian of hour angle** – (all points have same hour angle)

Observer meridian: $PZSQ$

- **Transit** – when an object cross this meridian
- HA measures westwards from observer meridian

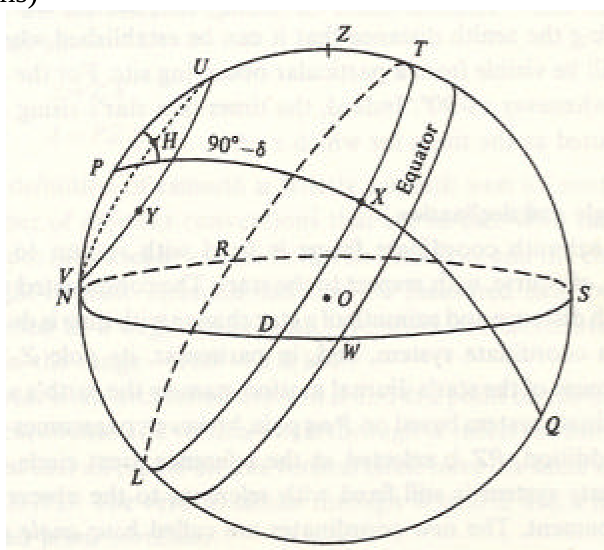
Parallel of declination = Small circle with poles PQ

- Diurnal motion of object along parallel of declination
- Progression westward from X to D (rising point R and setting point D)
- HA increases in clock wise direction about P (left-handed system)
    - o Azimuth west HA between 0h to 12h
    - o Azimuth east HA 12h to 24h

An object on celestial equator rises at East point and set at West point

- Stay above horizon 12h
- Negative d $\Rightarrow$ stays above horizon less than 12h
- Positive d $\Rightarrow$ stays above horizon more than 12h

NOTE: declination of stars = nearly constant – but not of the Sun which varies ± 23.5 degrees (seasons)



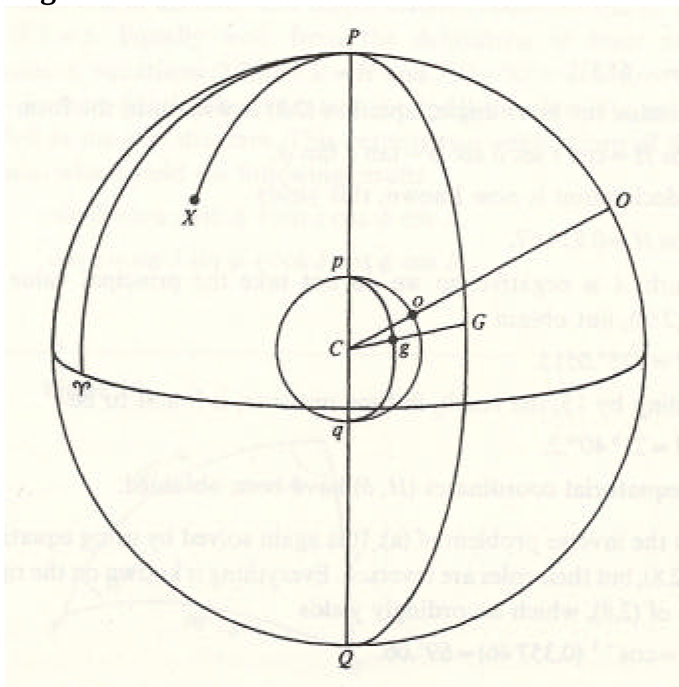
For Northern latitudes

- **Circumpolar objects** – always above the horizon $d > 90^\circ - f$
- **Never visible**: $-d > 90^\circ - f$

Transformation equations:

- $\sin d = \cos z \sin f + \sin z \cos f \cos A$
- $\cos z = \sin d \sin f + \cos d \cos f \cos H$

Sidereal time and right ascension



- C = centre of Earth – p and q = north and south poles
- P and Q north and south celestial poles
- g = position of Greenwich – projection on celestial sphere G = zenith point at Greenwich $\Rightarrow PGQ$ Greenwich meridian
- o = position of observer at east longitude l - projection on celestial sphere O = zenith point of observer $\Rightarrow POQ$ local meridian of observer at spherical angle $GPO = l$
- If X position of star – **Greenwich hour angle** $GHA = GPX$
- Hour angle of observer $HA = OPX = GHA + l$ (l expressed in time units)
- $a = \Upsilon PX$ is the right ascension of the star
- Υ also a reference for time = **Sidereal time**
- Local sidereal time $LST = HA\Upsilon$
- Greenwich sidereal time $GST = GAH\Upsilon$
- The two related by equation: $LST = GST + l$
- Sidereal day = 24hours = precisely one rotation of Earth about its axis (23h56m solar time – since Υ fixed while Sun moves relative to stellar background
- **Relation with right ascension**: $LST = HAX + RAX$

Example:

Radio telescope at longitude $l = 83^\circ 31' W$ and latitude $f = 40^\circ 15' N$

Radio source 3C273 with coordinates: $a = 12^h 28^m.3$ and $d = 2^\circ 08'$

Date of observation: 1985, January 7 at $14^h 42^m$ UT

From the Astronomical Almanac: for 1985, January 7 at 0^h UT $GST = 7^h 06^m 01^s$

1- Convert $14^h 42^m$ UT in sidereal time – express interval in hours and apply conversion factor 1.0027379094 $\Rightarrow GST = 21^h.8406$
2- Derive LST subtracting west longitude $\Rightarrow LST = 16^h 16^m 22^s$
3- Compute hour angle using $HAX = LST - RAX$ which yields $3^h 48^m 04^s = 57^\circ.0174$
4- Compute altitude $\sin a = \sin d \sin f + \cos d \cos f \cos AHX$ and azimuth $\cos A = \sin d \sec a \sec f - \tan a \tan f$ which yield $a = 26^\circ.0564$ and since hour angle is less than 12^h, the source azimuth is west $A = 111^\circ.0779$