IMF, SFR and stellar depletion in the local Galactic plane, based on improved Hipparcos samples of single stars

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ABSTRACT
We have used the recently reduced Hipparcos data and the Washington Double Star catalogue to derive two volume- and magnitude-limited samples of single stars in the Galactic plane. The star count method has been used to compare synthetic to empirical samples of stars within both the main sequence and the evolved regions of the Hertzsprung–Russell diagram. Corrections have been made for unrecognized binaries and the residual incompleteness of the Hipparcos catalogue. Two methods of binary reduction have been compared, a flat 71 per cent binary fraction (including multiples) and a mass-dependent binary fraction, which decreases with primary mass from 71 to 57 per cent.

A semi-empirical relation between the vertical scaleheight and stellar age has been derived from kinematic data provided by the OSACA data base. This relation is used to prescribe the loss of stellar content vertically from the plane due to heating.

When employing the fixed binary ratio, we find that the best fit is provided by a Scalo initial mass function (IMF) with an exponent of $1.85 \pm 0.15$, together with an average Galactic thin-disc star formation rate (SFR) of $618(\pm15)$ stars $\text{Myr}^{-1} \text{kpc}^{-3}$ with $1211(\pm30) \text{M}_\odot \text{Myr}^{-1} \text{kpc}^{-3}$ for single stars with $M_\star > 0.9 \text{M}_\odot$. The application of the mass-dependent variable binary ratio yields more single stars at lower mass and hence a steeper ($\Gamma = 2.2, \ldots, 2.3$) IMF and an increased SFR.

Key words: stars: evolution – stars: late-type – stars: luminosity function, mass function – Galaxy: disc – Galaxy: kinematics and dynamics – solar neighbourhood.

1 INTRODUCTION
Observations of local stellar kinematics show a clear trend of an increasing average velocity dispersion towards later spectral type, hence with increasing stellar age (Wielen 1977; Holberg, Nordström & Anderson 2007; Binney & Aumer 2009). An explanation of this observation is that stars were born on the Galactic plane with relatively small velocity components and have then been kinetically heated. This heating effect has been attributed to a number of processes: (i) encounters with massive interstellar clouds (Jenkins 1992), (ii) impacts with globular clusters (Vande Putte, Cropper & Ferreras 2009) and (iii) scattering by spiral arms (Binney & Tremaine 2008); although the case has not yet been resolved, each process results in an enlarged velocity dispersion. The amplification of the vertical velocity component causes the stellar thin-disc population to expand into the column, thereby depleting its number counts in the Galactic plane. For a detailed discussion, see Schröder & Pagel (2003).

It can be expected that (after thermalization) the observed vertical density scaleheight of the local galactic disc increases with age in proportion to the average vertical velocity dispersion – for a derivation, see Schröder & Pagel (2003). In other words, older stars are more widely distributed in the vertical direction, resulting from the increase in their energy associated with the vertical motion. Therefore, it is possible to study the long-term effects of vertical galactic disc dynamics and thus determine the initial mass function (IMF) and star formation rate (SFR) by means of simple star counts – provided that a complete volume-limited stellar sample is available, with well-defined age-specific subsamples. Presently, this approach comes with the disadvantage of small samples, but it has some unique advantages.

By contrast, classical work on the galactic IMF and SFR (e.g. Miller & Scalo 1979; Scalo 1986) used column-integrated star counts and therefore built on conveniently large samples. However, the method suffers from a number of uncertainties: a seriously compromised quality of the stellar data (without individual distance information), reliance on an approximate luminosity function (instead of using well-calibrated evolution tracks) and a poor ability to discriminate by stellar age.

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1.1 Data-mining the Hipparcos catalogue: earlier work on the solar neighbourhood

Earlier work (Schröder & Pagel 2003) concentrated on a complete volume-limited sample of single stars, derived from the Hipparcos catalogue (Perryman 2002). A cylindrical volume centred on the Sun was chosen with a radius of 100 pc and a height of 50 pc (i.e. $|z| \leq 25$ pc), within which the Hipparcos catalogue is nearly complete down to $M_v = 4.0$. All known astrometric and spectroscopic binaries were removed and some minor adjustment was made to number counts to correct for residual incompleteness in the faintest group of stars. The sample was divided and studied in terms of 10 well-defined subsamples: seven groups of main-sequence (MS) stars and three groups of evolved stars. For the present study, we have used the same field boundaries for the different MS stars, but have added one extra subsection [blue loop (BL)] to the evolved stars (see Fig. 1).

There is a key information extraction problem with traditional population synthesis studies (see e.g. Binney, Dehnen & Bertelli 2000): the degeneracy of the star counts along the MS with respect to the distribution of the initial masses (IMF) and a possible time dependence of the SFR. Any set of observed MS counts can be matched by a whole family of synthetic populations with different IMF and SFR($t$). For example, a shallower IMF can be compensated by a larger SFR in the more distant past to give the same present-day mass function (PDMF), observed by the current MS star counts.

There is a way to separate the age-related information from the one related to mass and age combined. For any random distribution of ages, a subsample of evolved stars is about twice as old as a sample of MS stars of the same average mass. For this very reason, special attention was given by Schröder & Pagel (2003) to reproducing the observed ratio of evolved to MS number counts. Each of these age- and mass-specific groups of stars is defined by occupying a specific field within the Hertzsprung–Russell diagram (HRD), defined by $M_V$ and $B – V$ boundaries. To match all star counts by population synthesis, evolved stars included, it is necessary to assume a local SFR increasing with time. Without this assumption, the synthetic number counts of the evolved stars turn out to be much too large (see Bertelli & Nasi 2001). This points to the aforementioned depletion of the older stars in the local neighbourhood by spreading into the column. In other words, the thin-disc SFR is determined by multiplying the local SFR by a depletion factor ($<1$) which increases with time; see Schröder & Pagel (2003) for a more detailed discussion.

In the same study we have shown that the time-scale for radial diffusion appears to be of the order of 0.7 billion years, about 10 times faster than the time-scale for vertical diffusion. For all older stars, radial mixing fully provides our solar neighbourhood (despite its inter-spiral arm location) with the average galactic thin-disc stellar population. Hence, looking at stars in the age range of 0.7–9 billion years, a local stellar sample fully represents the average galactic thin-disc IMF.

1.2 Motivation and main points of this study

Certainly, a quantum leap in this field will be generated with the availability of the Global Astrometric Interferometer for Astrophysics (GAIA) data. GAIA will provide accurate distances and other stellar properties within a sizeable portion of our galactic disc. However, these data are still a decade away and a number of recent advances make it possible to improve upon previous work today.

(i) The most noticeable advance came in the form of a much more complete catalogue of binary stars: the latest version (2006 December) of the Washington Double Star catalogue (see Mason et al. 2001; the most up-to-date version is available at http://ad.usno.navy.mil/wds/). From this data base, we find the local binary fraction of the stellar population (Section 2.2) and we are able to determine the corrections to the single-star counts for the biased content of unrecognized binaries, which increases both with distance and towards lower luminosities. We here test two assumptions: a flat ($f(r)$) or a variable ($v(r)$) binary fraction over the mass range under study ($>0.9 M_\odot$).
(ii) Other recent progress comes from an improved *Hipparcos* data reduction (Van Leeuwen 2007). This provides improved individual parallaxes and thus reduces the Malmquist-type bias (Malmquist 1936) towards larger distances. This important improvement enables the volume of study to be expanded, thus increasing the number of evolved stars and reducing the statistical uncertainty.

(iii) In addition, more emphasis has been placed on applying an accurate residual incompleteness correction for the *Hipparcos* samples (see Section 2.3). The corrections for incompleteness (positive) and for unrecognized binary content (negative) almost neutralize each other in their effect on the empirical, ‘true’ single-star counts. Hence, the new empirical star counts for the $r = 100$ pc sample do not differ too much from the ones presented by Schröder & Pagel (2003), but they are now considered less biased.

(iv) The last advance comes from improved synthetic population models. Radial velocity data from the OSACA data base (Bobylev, Gontcharov & Bajkova 2006) for our stellar subsamples have been used to derive an empirical velocity–age relation (see Section 3). From this, a semi-empirical scaleheight–age relation has been determined (calibrated from star counts over a range of subsamples; see Section 4). This enables the introduction of a semi-empirical prescription of the in-plane depletion of evolved stars. This in turn reduces the number of input parameters, ensuring that the population synthesis models can concentrate on recovering the IMF and SFR.

These improvements allow for the study of a more reliable thin-disc stellar sample within 100 pc radius as well as for a larger sample (within 150 pc radius from the Sun). The latter, while already limited in its accuracy, offers larger number counts for the rare bright stars, i.e. massive MS stars and BL giant stars.

## 2 VOLUME-LIMITED SAMPLES OF SINGLE STARS FROM THE *HIPPARCOS* CATALOGUE

Any interpretation of the stellar distribution in the HRD by means of synthetic populations based on evolution tracks for single stars can be confused severely by the unrecognized presence of binary systems in the observed samples (Bertelli & Nasi 2001; Cignoni et al. 2006). The presence of a companion star changes the HRD position in luminosity and colour and thus makes the star useless for fitting. Much attention was paid to the removal of all binaries from the samples.

Close bright binaries and binaries with an almost equally bright companion are more readily identified, and an almost complete sample of these systems exists for the solar neighbourhood because their nature is revealed by high-resolution spectroscopy, thanks to the Doppler effect. However, many binaries containing a faint companion remain unrecognized, especially towards lower apparent brightness. Quist & Lindegren (2000) studied these stars within the *Hipparcos* sample by means of computer simulations. They found a maximum possible multiplicity fraction of 83 per cent, suggesting that our sample is indeed missing such binaries. Fortunately, for these systems the primary star’s position in the HRD is not greatly affected. The distribution in the HRD and the IMF of the primaries in the local binary star population in fact resembles that of single stars (see Fisher, Schröder & Smith 2005). Hence, unidentified binaries existing within a single-star sample might not be regarded as critical. However, there remains an important problem: the fraction of recognized binaries systematically decreases towards larger distances and lower luminosities. This bias, in combination with the large binary fraction, leads to systematic distance- and luminosity-related effects. For this reason, a study of both the true binary content and the residual incompleteness has been completed in detail and then separate corrections for our observed star counts have been applied on a statistical basis.

### 2.1 The initial sample

An initial volume-limited sample with an outer boundary at a radius of 150 pc was created. Consisting of 39 970 stars listed in the *Hipparcos* catalogue, this sample is centred on the Sun and so an offset of 15 pc to the north of the Galactic plane (Cohen 1995) has been applied. For cross-referencing, we used a binary star catalogue of 11 326 entries, primarily consisting of stars from the most recent Washington Double Star catalogue, complemented by photometric binaries.

In order to quantify both the true binary content and the residual incompleteness of the initial sample, three subsamples of different sizes were created. All three are confined to $|z| < 25$ pc and $M_z < 4$, but they differ in their radii, which are 65, 100 and 150 pc, respectively, in order to create three cylindrical slabs of different radial extent in the Galactic plane. These samples were then studied in terms of 11 mass- and age-sensitive star groups; seven cover consecutive ranges on the MS and four characterize different evolved stages in the HRD (see Fig. 1). Without any distance-related biases, we should expect all specific star counts to grow approximately with the radius squared. Hence, for each of our star groups, we can quantify the distance-related biases, unrecognized binary fraction and incompleteness on a statistical basis, by comparing the respective counts from the above three subsamples. The raw counts are given in Table 1.

### 2.2 Corrections for the unrecognized binary content

Studies have been inconclusive about the local binary fraction: Lada (2006) suggests a variable fraction which drops towards lower mass stars, whilst work completed by Fisher et al. (2005) suggests a

### Table 1. Raw counts of apparent single stars and known binary stars, specified for the 11 star groups shown in Fig. 1 and for three Galactic plane samples of different radial extent (65, 100, 150 pc), each with $|z| \leq 25$ pc.

<table>
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<tr>
<th>Sample</th>
<th>MS1</th>
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<td>65 pc, app. single stars</td>
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<td>known binaries</td>
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<td>known binaries</td>
<td>15</td>
<td>66</td>
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<td>207</td>
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The fraction of known binaries in the initial sample decreases considerably towards lower luminosity, along the MS (our star groups 1 to 7), as well as for the giants (by increasing luminosity: 8 = LXB, 9 = KGC, 10 = BL giants). Among the three subsamples, the known binary fraction decreases with a larger mean distance. We adopt a maximum true binary fraction of 0.71, here used to correct the apparent single-star counts by the corresponding numbers of unrecognized binaries.

constant fraction, since the slopes of the IMFs of primaries and single stars are nearly identical. While Fisher et al. (2005) study the same mass range as used in this sample (i.e. $M_\ast > 0.9 M_\odot$), most of Lada’s arguments apply to much lower primary masses. In any case, there are large discrepancies between the fraction of known binaries and the true fraction, which give rise to a lot of speculation and dispute about the latter. The fraction of unrecognized binaries depends upon the distance, luminosity and other binary properties.

In our sample, the fraction of known binaries (including multiple systems) evidently decreases with both radial extent and lower average luminosity of the respective star groups, as shown in Fig. 2. While the majority of the subsamples has a known binary fraction of around 40 per cent, by contrast its value reaches over 70 per cent in the brightest MS and giant star groups (highest values in the absolute brightest groups of the smallest radial extent), which contain the best-studied stars. Hence, the lower fractions of known binaries in the other groups could entirely stem from incomplete binary detection rates there. But, in part, there may also be a real drop with decreasing mass.

If we assume that the known binary fraction equals the true one for the brightest stars (MS1) within the smallest subsample (radial extent of 65 pc), we arrive at a fraction (including multiples) of 71 ± 22 per cent. This value is well replicated by the more statistically significant 100 pc sample where a value of 68 ± 15 per cent is observed. The more or less constant 40 per cent detection rate within the 150 pc sample is thought to represent the base line detection rate for binary systems.

While the binary fraction appears larger than commonly expected, it is in good agreement with recent work (Quist & Linddegren 2000; Converse & Stahler 2008; Eldridge, Izzard & Tout 2008). In fact, the constant increase in data quality has, for decades now, led to an ever upwards correction of the commonly assumed binary fraction. Counter to this trend, Lada (2006) argues for there still being a majority of single stars, when including stars down to brown dwarf mass. For stars about as massive as the Sun, he adopts a fraction of 54 per cent, citing Duquennoy & Mayor (1991) for their study of spectroscopic binaries in a complete sample of G-stars. This classical study results in 57 per cent of binaries including multiples, but it did not include astrometric binaries, which we also count as known binaries – their drop in numbers towards fainter objects is a major concern for correcting the apparent single-star counts. Hence, if wide astrometric binaries had been included, the Duquennoy & Mayor (1991) value might have come quite close to the 71 per cent which we find as the maximum fraction of known binaries. We therefore believe, while Lada’s arguments for a decreasing binary fraction do apply to the mass range below $\approx 1 M_\odot$, that above this mass the binary fraction has already levelled out. Hence, our preferred method to correct the counts of apparent single stars for unrecognized binaries is based on a flat binary fraction ($f_\mathrm{r}$) of 71 per cent.

As an alternative, we also test a correction which is based on the maximum decrease allowed by the empirical evidence discussed above: from 71 per cent, for the most massive stars (groups MS1 and BL giants), down to 57 per cent (including multiples; Duquennoy & Mayor 1991 for G-stars), for MS7 (just over 1 M$_\odot$). The rates assumed for the other subsamples are as follows – MS2: 68 per cent, MS3: 65 per cent, MS4: 63 per cent, MS5: 61 per cent, MS6: 59 per cent, lower giant branch (LGB): 57 per cent (mostly evolving from MS7 stars), K-giant clump (KGC): 62 per cent (mostly evolving from MS4 and MS5 stars) and cool wind (CW) giants: 65 per cent (mostly evolving from MS2–MS4 stars).

By comparing the fraction of known binaries with a certain adopted true binary fraction, it is possible to determine the number of unrecognized binaries (on a statistical basis) and subtract it from each count of apparent single stars.

### 2.3 Corrections for incompleteness of the Hipparcos catalogue

The maximum extent of 150 pc is not large enough to encompass any variations in the radial density of the spiral structure. It can then be assumed that there exist very similar (true) stellar densities within all three subsamples. Any shortfall in the least-luminous star groups in the farthest reaching subsample therefore indicates and quantifies the residual incompleteness of the Hipparcos catalogue, rather than being real. Fig. 3 plots the three subsamples’ stellar densities over MS star groups 1 to 7, and the four groups of evolved stars (see Fig. 1) are labelled 8 to 11.

Fig. 3 shows that densities compare favourably between the three subsamples, with the only two exceptions being the faintest MS groups 6 and 7. We have corrected the deficiency in star numbers in these groups by the factors by which their densities fall short of the 65 pc subsample. Accordingly, we added 20 per cent to the count of MS7 in the 100 pc sample and both 76 per cent to MS7 and 25 per cent to MS6 in the 150 pc sample, assuming that all...
incompletenesses are mainly of a statistical, and not systematic, nature. The resulting ‘true’ single-star counts, in addition corrected for unrecognized binaries and incompleteness, are given in Table 2. Note that the incompleteness corrections themselves are independent of the binary corrections (either \(fr\) or \(vr\)) since the binary corrections cancel in the density ratios used for the incompleteness corrections.

### 3 OBSERVED STELLAR KINEMATICS AND SCALEHEIGHTS

A semi-empirical relationship between age \(\tau\) and scaleheight \(H_z\) was determined in order to better constrain the depletion of stars from the plane due to disc heating.

The scaleheight was constrained using off-plane subsamples with \(25 < |z| < 50\) pc and \(75 < |z| < 100\) pc, which span an effective \(z\)-difference of 49 pc. The respective slab volumes have a ratio of 1.423:1, by which the corrected and listed number counts are divided in order to get stellar number densities \(\rho_i\). The empirical scaleheight is then given by \(H_z = 49\ \text{pc}/\ln(\rho_{25,50}/\rho_{75,100})\). Due to the small number statistics, we have sufficient confidence only in a joint scaleheight derived from the integrated star counts of MS groups 3 to 5, which together represent an average age of 1.04 Gyr. For these, we derive mean scaleheights of 253 pc (\(fr\)) and 255 pc (\(vr\)) – which are essentially the same value, independent of the corrections made for undetected binaries. Note that this scaleheight may only apply at small vertical distances from the Galactic plane (<100 pc).

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However, these regions are vital for modelling the depletion of stars within our sample.
The Hipparcos sample is unable to provide a precise relationship between age and scaleheight solely by number counts, for two reasons. First, the volume studied is small compared to the scaleheights, while secondly the variation in the ratios of the number counts in adjacent subsamples is not much greater than expected by natural variation. The purpose of the previously derived scaleheight is therefore to calibrate the more accurate age–scaleheight relation that has been determined from velocity data.

The OSACA data base (Bobylev et al. 2006) combines, as far as available, radial velocity data with the Hipparcos proper motions and parallaxes and readily provides the resulting stellar space velocities in galactic coordinates. We used the available OSACA vertical velocities of all stars in our spherical 150 pc sample and added a vertical solar velocity of $+7.17 \text{ km s}^{-1}$ (Dehnen & Binney 1998), to get the absolute vertical velocities $\sigma_z$ of each star relative to the Galactic plane. From these, we derived for each of our specific star groups (according to Fig. 1) the vertical velocity dispersion (i.e. the rms of vertical velocities) as a function of theoretical average age (see Table 2) of the respective star groups.

Fig. 4 shows that there is a clear increase in the average vertical velocity dispersion with the average age $\tau$ for the different star groups within our sample. This increase in velocity dispersion can be modelled by the following equation:

$$\sigma_z = 10.1 (1 + \tau/\text{Gyr})^{0.45} \text{ km s}^{-1}.$$  

The exponent of 0.45 is very close to that found in the classical work (Wielen 1977) and aligns well with the current work (Binney & Aumer 2009). This value re-iterates that heating cannot be caused solely by interactions with molecular clouds (Hänninen & Flynn 2002), where an exponent of 0.26 is required.

While this relation is not linear and implies over the large age ranges of MS subgroups that the average velocity dispersion is not exactly identical to the velocity dispersion of the average age given by the derived relation, this systematic second-order error is small when compared to the statistical uncertainties in the measured $\sigma_z$ values. In fact, the curvature (see Fig. 4) over each age range is relatively small, while the statistical errors and the variation against the derived function are of the order of 10–20 per cent.

A comparison to other $\sigma_z$ values (e.g. those obtained by Binney & Aumer 2009 for their significantly larger sample) is difficult, since stars have been grouped differently. For example, the above named study distinguishes the stars by the $B-V$ colour only, not directly by age. However, our velocity dispersions also appear to saturate at somewhere between 3 Gyr (as found by Quillen & Garnett 2001) and 5 Gyr (Binney & Aumer suggest saturation after 4 Gyr). While the larger scatter in Fig. 4 shows that our more restricted sample is smaller, we believe that our approach still has an advantage (less quantity, but more quality): the Malmquist bias should be smaller, for young stars in particular. This is because many of the bright stars in the Binney & Aumer (2009) sample lie outside our volume of study and hence have only marginal parallaxes. Since parallax errors are asymmetric in distance (much like errors in the distance modulus; see Malmquist 1936), a statistical shift towards smaller distances results, which artificially produces a smaller velocity dispersion (via smaller transversal velocity components derived from the proper motion data). In addition, reducing our sample by the known binary stars may also make a difference. As shown in Table 2, $\sigma_z$ values of binary stars differ from those of single stars in several cases.

Assuming a proportionality between average scaleheights and vertical velocity dispersions as functions of average age (see Schröder & Pagel 2003), and using our empirical direct measure of an average scaleheight of 254 pc (for the MS3 to MS5 stars with about 1.04 Gyr of average age; see above) to scale the above equation into a scaleheight–age relation, we finally derive the following semi-empirical model of the vertical thin-disc expansion with age:

$$H_z = 184 (1 + \tau/\text{Gyr})^{0.45} \text{ pc}.$$  

In the following section, we use this relation to model the stellar long-term depletion on the Galactic plane by means of a dynamical dilution. According to the above equation, vertical scaleheights expand by more than a factor of 2, from 234 pc, for stars with an age of about 0.7 Gyr, to about 500 pc, for its oldest stars (8–9 Gyr). The latter value may appear large in terms of thin-disc stars, but it agrees exactly with the empirical scaleheight found by Jura & Kleinmann (1992a,b) for low-mass Mira stars with an estimated age of 8–9 Gyr, characterized by periods of 200–300 d.

At the other extreme of age, stars younger than 0.7 Gyr have scaleheights much smaller (see Table 2, under MS1 and MS2) than suggested by the above relation. But these stars are still in the process of radial mixing, by which they arrive in our inter-spiral-arm region with zero star formation. Hence, their counts do not reflect the vertical long-term depletion processes, and their initial scaleheights reflect those of the gas clouds from which they were born. In Schröder & Pagel (2003), we suggested that their scaleheights grow rapidly with age as these stars move towards dynamical equilibrium.
4 MODELLING THE STELLAR POPULATION AND ITS DEPLETION IN THE GALACTIC PLANE

4.1 The population synthesis model

The only input parameters required by our synthetic thin-disc single-star populations in the Galactic plane are the IMF and the average SFR. We also use the previously determined depletion description, as opposed to trial and error (Schröder & Pagel 2003). By contrast, column-integrated studies of the galactic thin disc (Miller & Scalo 1979; Scalo 1986) recover the IMF and average thin-disc SFR without having to consider depletion in the Galactic plane. It appears from such studies that over larger time and spatial scales, the average thin-disc SFR is fairly constant. This makes it easier to verify a certain long-term depletion with age by the type of approach presented in this paper.

The best match between synthetic and real stellar populations is defined by how well the corrected empirical single-star counts in all our age- and mass-specific groups (see Table 2) are reproduced. Each population model is created by generating a random distribution of masses (with a mass-dependent probability obeying a prescribed IMF) and ages. Those synthetic stars which survive the age-dependent depletion (again by random choice) are placed in the HRD according to a fine-meshed grid of well-calibrated evolution tracks. The evolution code was originally developed by Eggleton, Pols et al. (1997, 1998) and Schröder, Pols & Eggleton (1997). In the grid used in this study, overshooting sets in for $M_\odot > 1.55 M_\odot$. We find that this grid gives the most consistent reproduction of the star counts along the MS, in close agreement with our earlier study (Schröder & Pagel 2003).

4.2 The equivalence of modelling processes

Earlier work (Schröder & Pagel 2003) also showed that the long-term depletion of older stars in the Galactic plane can be modelled in two ways: (i) by a simple diffusion approximation, with a common diffusion time, $\tau_D$, for all stars as an adjustable parameter, or (ii) by prescribing (i.e. without an adjustable parameter) a diffusion proportional to the increase in the vertical stellar density scaleheights with age as previously determined. In principle, this equivalence should not come as a surprise. In a strictly exponential vertical density distribution $n(z)$, the gradient $dn/dz$ is inversely proportional to the scaleheight $H_z$. Any diffusion time $\tau_D$, on the other hand, is proportional to the product of the density gradient and the average vertical stellar velocity dispersion $\sigma_z$; hence $\tau_D \propto \sigma_z / H_z$. Since (in the diffusion model) $\sigma_z$ and $H_z$ depend on stellar age in the same way (see the previous section), any age dependence of the diffusion time cancels out in a first-order approximation.

In reality, of course, the vertical density distribution is not exactly exponential. Hence, the diffusion approach should be more accurate, since it is derived directly from galactic kinematics. Even more important is that it avoids adding an adjustable parameter to the modelling process (while the diffusion approximation does). Hence, the population synthesis study can concentrate on finding the correct IMF and SFR.

4.3 Quantification of the quality of a match

To quantify the quality of a match of a particular synthetic population to the empirical star counts, we use a weighted mean $\langle \sigma \rangle$ of the differences $\Delta N_i$ between the synthetic counts and the observed counts $N_i$ (within the specific star groups shown in Fig. 1), each divided by $\sqrt{N_i}$ (see also Schröder & Pagel 2003). In addition, we give twice the normal weight to the large giant counts (LGB and KGC). As a result of this, and also because some star counts are strongly correlated and are not statistically speaking fully independent of each other (e.g. the LGB giants and MS7 counts), a weighted mean difference of about 0.5 already becomes statistically significant (i.e. is comparable to a conventional $1 \sigma$ deviation).

In order to avoid the additional statistical variation of the synthetic sample counts, but to deal only with the number statistics of the empirical star counts, we computed inflated synthetic samples (up to 30 times for matches to the 100 pc sample). For the 100 pc samples, we considered the range of models matching better than $\langle \sigma \rangle = 0.5$ as indicative of the uncertainty margin, i.e. $\pm 0.15$ in $\Gamma$ and about 2.5 per cent in the SFR. In the 150 pc samples, best fits reach only $\langle \sigma \rangle \lesssim 1$, for which we believe that two reasons are contributing: (i) with decreasing variation by number statistics, which is built into the weighting factors, physical population variations (e.g. by star clusters) start to make a larger contribution to $\langle \sigma \rangle$; (ii) the corrections made to the empirical single-star counts become too large for the 150 pc sample and contribute to mismatches.

Our empirical in-plane samples of radii of 100 and 150 pc (both with $|z| \leq 25$ pc) have volumes of $1.57 \times 10^6$ and $3.53 \times 10^3$ pc$^3$, respectively. The slope of the IMF (our exponent $\Gamma$) follows the Scalo-type definition: $dn_*/d \log M_* \propto M_*^{-\Gamma}$. Where possible, we gave preference to matching populations with just one straight slope, although our models allow for a bend at 1.66 $M_\odot$ (MS stars: age of 0.7 Gyr). This is where the local IMF of younger stars might be shaped by insufficient radial mixing into the solar neighbourhood.

4.4 Results

Table 3 summarizes and compares the best-matching synthetic populations with the empirical counts. In the case of the 100 pc sample and a (preferred) flat binary ratio ($f_r$ of 71 $\pm 22$ per cent), we find an IMF exponent of $1.85 \pm 0.15$ – within the error bars very close to the classical value of 1.7 (Miller & Scalo 1979; Kroupa, Tout & Gilmore 1993) – and an average galactic thin-disc SFR of 618($\pm 30$) per kpc$^2$. The IMF is given in these units depends, in addition, on the exact fraction of binary systems in the relevant mass range and on their average mass ratio (which we did not study here). For an average mass ratio of the order of

Our depletion by the dilution prescription applies to the age range of 0.7–9 Gyr. The counts of younger stars are dominated by radial mixing, and we consider here only marginal ($< 0.1$ per cent, $\tau_D = 6.3$ Gyr) vertical diffusion losses. The population models presented below prove that our long-term depletion prescription leads to the best possible matches of the empirical star counts. It is nearly equivalent to a simple (but kinematically incorrect) diffusion description with a general vertical diffusion time $\tau_D = 6.3$ Gyr.
0.7 (secondary/primary), the total SFR would be about four times larger in units of $M_{\odot}$ Myr$^{-1}$ kpc$^{-3}$ than the single-star SFR alone.

The $v_r$ assumption has more single stars at lower mass since fewer discounts are made for unknown binaries. Hence, a steeper ($\Gamma = 2.2, 2.3$) IMF is required and a 35 per cent larger SFR. Similarly, the incompleteness corrections made for the 150 pc samples require an even steeper IMF for the lower masses. We suspect that this reflects over-corrections made: because of the relatively small effective mass range, a modest over-correction can lead to a relatively large effect on $\Gamma$. This shows that beyond 100 pc, in the stringent terms of completeness and knowledge of the binary ratio, Hipparcos data have become too limited.

In all cases studied here, the prescribed (i.e. not adjustable) long-term, in-plane depletion by dilution gives the best possible matches of all empirical star counts. Over all, this depletion is nearly equivalent to a simple vertical dilution description with a general time-scale of 6.3 billion years – except that the latter consistently produces a slightly smaller number of LGB giants (the oldest subsample). In the extreme case of not considering any depletion (see our no-depletion model for the 100 pc sample in Table 3), the synthetic counts of the lower MS, and even more so of the oldest giants (LGB), increase. Hence, the synthetic ratio MS7/LGB decreases considerably, in this case from 9.6 to 8.0 (observed: 9.5). If only MS counts were matched this way, while ignoring the evolved counts, such a population model would suggest a much lower $\Gamma \approx 1$; (see e.g. Sabas 1997). For this reason, a semi-empirical, long-term depletion description is paramount to a population study of a local, in-plane stellar sample. At the same time, we find that the physical thin-disc SFR itself (before depletion) must have been reasonably constant, averaged over the whole thin disc and long intervals.

From our matching synthetic population (100 pc sample), we find an average stellar age of 2.35 Gyr. According to our $H_0$ (age) relation in the depletion by the dilution description, this corresponds to an average scaleheight of 317 pc. This value is in good agreement with the conical thin-disc scaleheight of 300 pc (Gilmore & Reid 1983). Note that it was derived for small vertical distances from the Galactic plane (see the previous section). Further out, the vertical thin-disc density slope appears to steepen to a scaleheight of about 250 pc, as studies with a larger sample volume indicate (e.g. Kuijken & Gilmore 1989; Vallenari et al. 2006).

### 5 CONCLUSIONS AND OUTLOOK

With this study, we have been able to produce a synthetic population of single stars, which matches the local empirical sample and successfully uses a semi-empirically derived depletion by the dilution model for the older stars. By contrast to earlier work, we now avoid the use of an adjustable parameter (a vertical diffusion time-scale).

A major problem of the single-star approach, which we have attempted to resolve in two different ways, is the removal of the bias from empirical single-star counts which arises from the biased fraction of unrecognized binary stars in different subsamples. In addition, we used the age-dependent information embedded within the counts of evolved stars in order to avoid pseudo-matches of the MS star counts by e.g. an SFR increasing with age (producing more, old low-mass MS stars) together with a flatter IMF (fewer low-mass stars born). In this way, we solve the common problem with the age-mass degeneracy in the information content of the star counts on the MS alone. As a result, we find a reasonably constant (in the long-term) SFR of $618(\pm15)$ stars Myr$^{-1}$ kpc$^{-3}$ (i.e. single stars with $M_* > 0.9 M_{\odot}$) and an IMF with a slope ($\Gamma = 1.85(\pm0.15)$) very close to the classical value of Miller & Scalo (1979, $\Gamma = 1.70$), which was derived from a column-integrated study.

Our present study also shows two remaining limitations with the present-day data: (i) the Hipparcos limitation in brightness leads to severe incompleteness beyond 100 pc above a stellar luminosity of $M_V = 4.0$, which is not easily corrected for; (ii) there still remains a stark discrepancy between the numbers of known binaries, which are strongly biased with distance and brightness, and the true binary content. Corrections to the star counts for the different content of unrecognized binaries in the different subsamples depend upon how the true binary fraction is regarded. We believe, from our study of the biases of the known binary fraction, that in the mass range studied here, above about 1 $M_{\odot}$, the true binary fraction (including multiple systems) has already, more or less, levelled out to a flat, average value of 71 per cent, leaving only 29 per cent of all systems...
to be single stars. A binary fraction that varies with mass between 71 and 57 per cent, which is the maximum variation permitted by the present empirical evidence, leads to a population of remaining single stars with a considerably steeper IMF (Γ between 2.2 and 2.3). Hence, the practical benefit from next-generation stellar data bases (GAIA, in particular) will need to be that they provide both a much larger sample size (increased spatial extent) and also a much better known binary content.

The synthetic stellar population presented here, together with its semi-empirically derived and tested depletion prescription, will lead us to a model of the thin-disc white dwarf (WD) population (work in progress; see also Schröder, Pauli & Napiwotzki 2004). It represents the ‘stellar graveyard’ of the thin disc, in which is buried additional information about the early galactic star formation history and the age of the thin galactic disc. This is even more significant, since GAIA data will eventually also provide us with a sufficiently large, empirical, complete (volume-limited) sample of WDs in the solar neighbourhood. Since WDs are, on average, more than twice as old as the sample of visible stars discussed here, a correct depletion prescription is even more important for modelling their population.

Another interesting application of an accurate, synthetic thin-disc population, like the one presented here, lies in the combination with a catalogue of stellar spectra (e.g. STELIB; Le Borgne et al. 2003). The synthetic spectrum of the whole population can then be computed. If the problem of interstellar absorption can be accounted for in sufficient detail, i.e. for different inclinations of the galactic disc with respect to an extragalactic observer, quantitative and direct spectroscopic comparison between other galaxy’s discs and our own can finally be made.

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